

# $E\{Z\}$ -Kriging

Exploring the World of Ordinary Kriging

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July 2004 (version 0.2)

# What is $E\{Z\}$ -Kriging?

- a computer program for exploring ordinary kriging;
- interactive, self-explanatory, and easy to use.  
Hence, 'easy kriging'
- and last but not least: it's freeware!

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- 1 Students, who need to understand those 'mysterious' kriging equations;
- 2 Lecturers, who want to explain kriging in an intuitive way;
- 3 Others, who just want to know what they are doing when using geostatistical software.

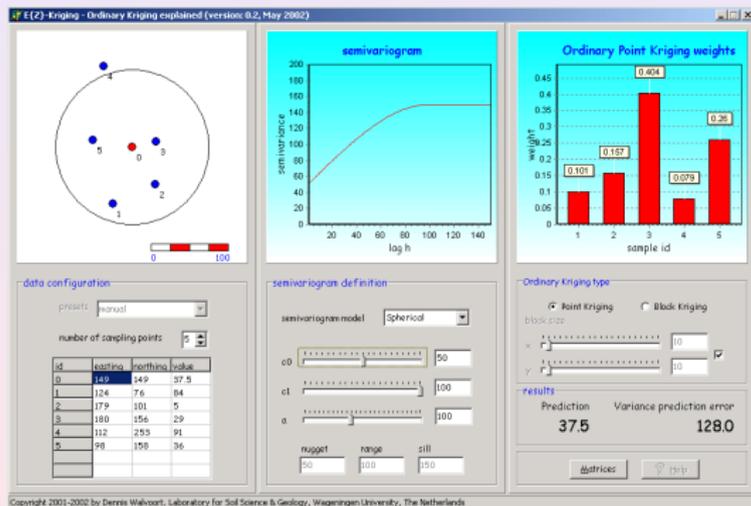
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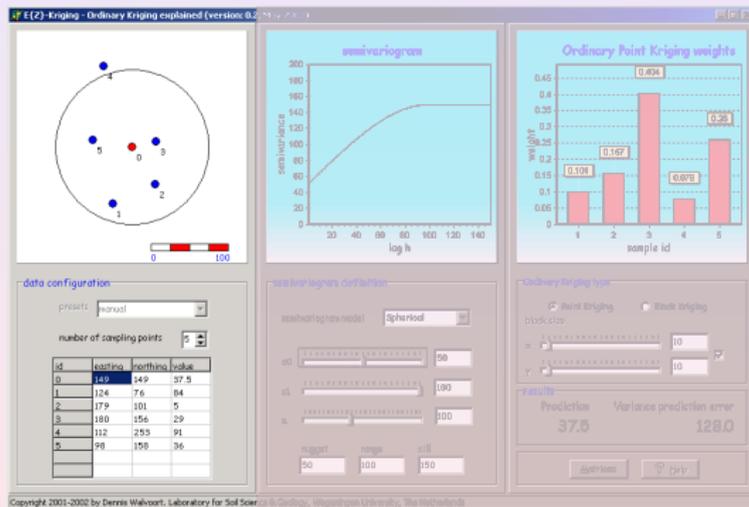
# Graphical User Interface (GUI)



The GUI consists of three panels:

- 1 data configuration panel
- 2 semivariogram panel
- 3 kriging panel

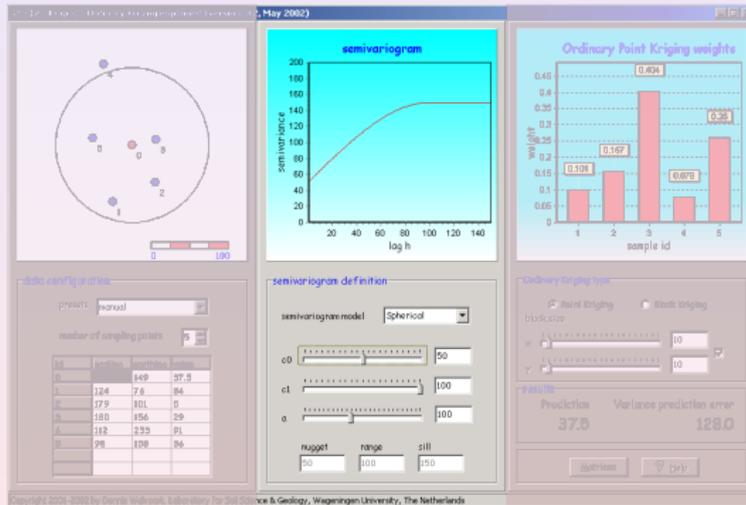
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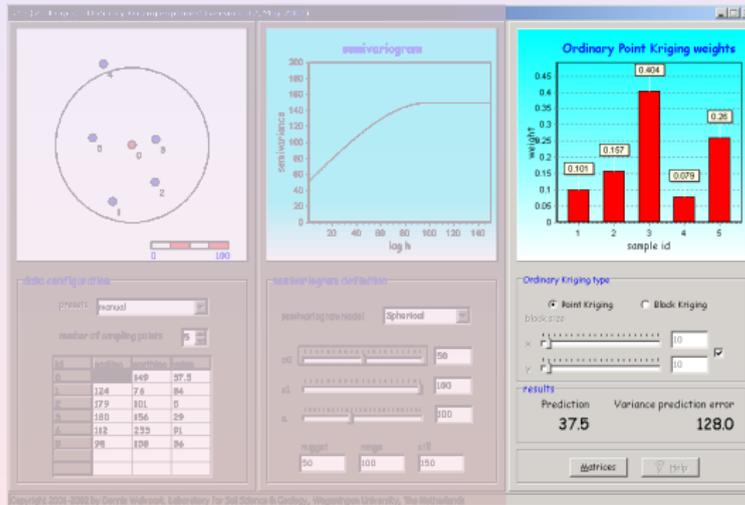
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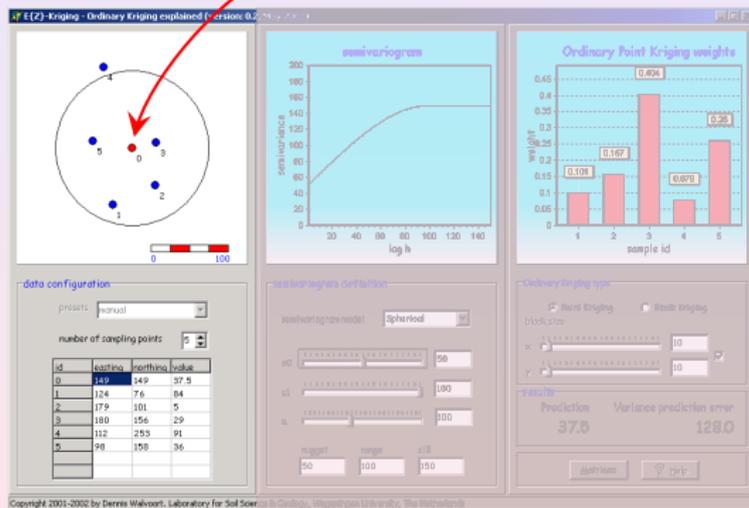
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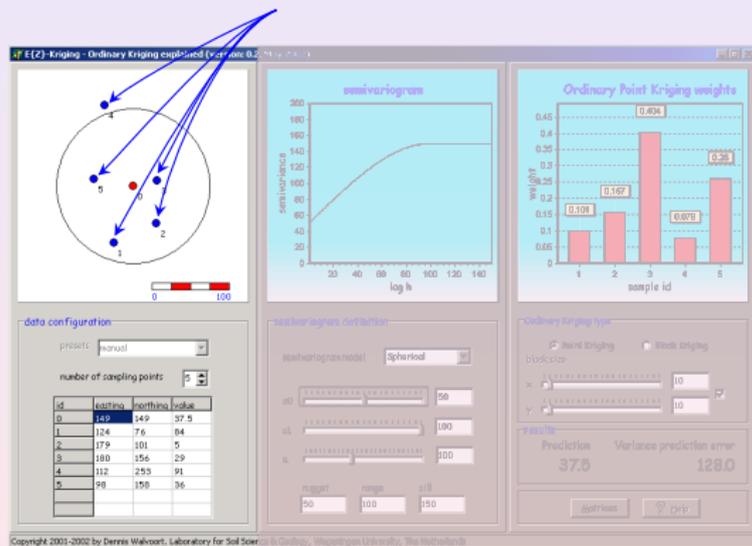
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# Data Configuration Panel



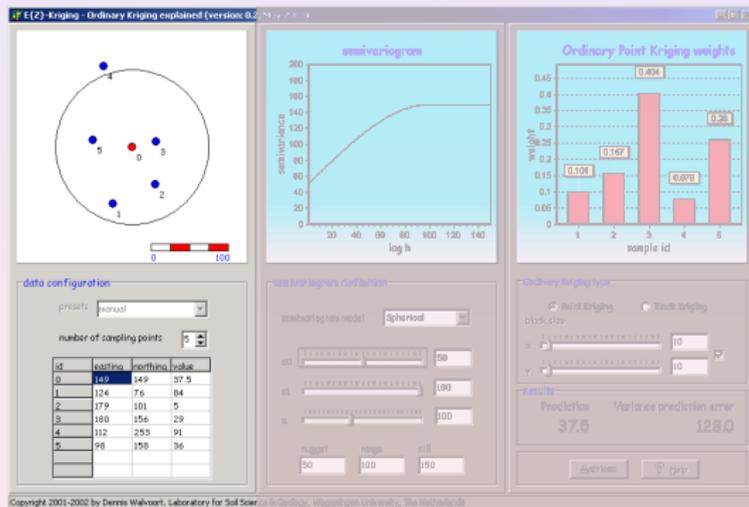
The prediction point is **red** and labelled 0.

# Data Configuration Panel



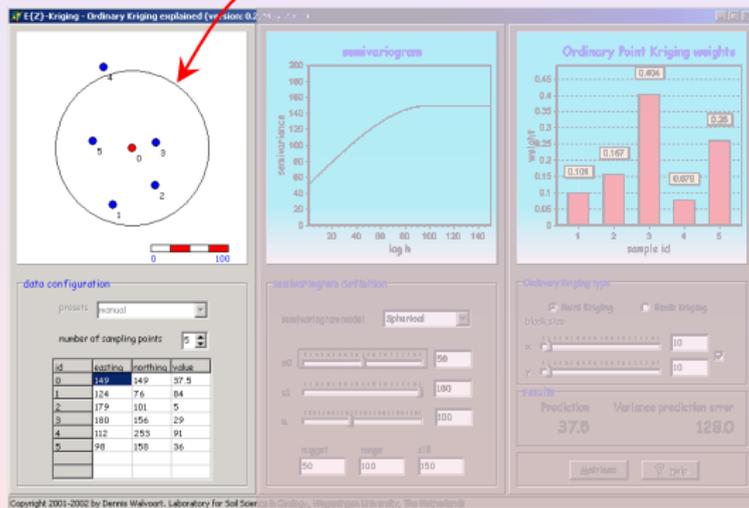
The  $n$  sampling points are blue and labelled from 1 to  $n$ .

# Data Configuration Panel



The configuration of points can be changed by means of the mouse (*drag 'n' drop*).

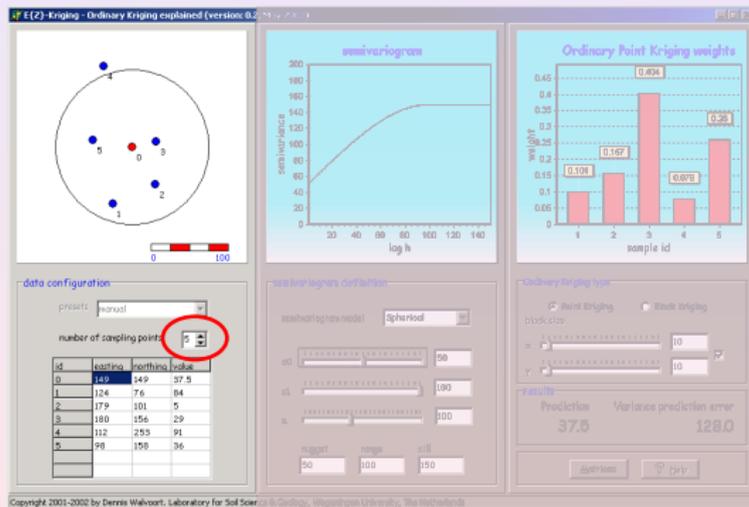
# Data Configuration Panel



The range is represented by a solid circle. A dashed circle represents the *practical* range.

► definition

# Data Configuration Panel



The number of sampling points has to be specified here (see ellipse).

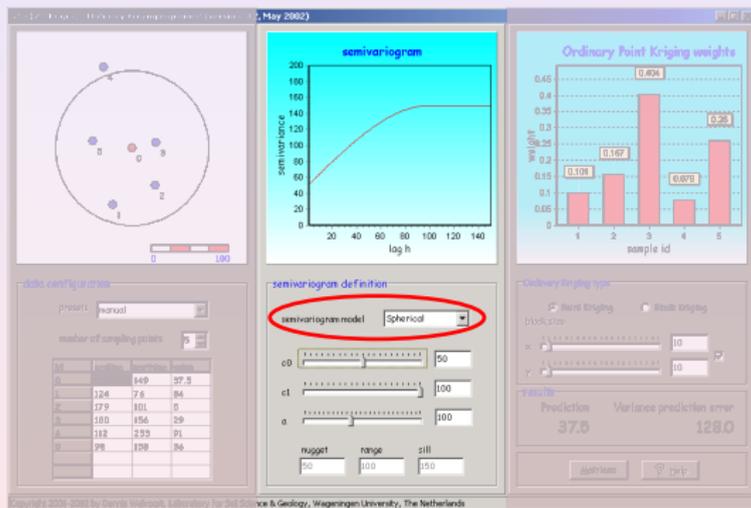
# Data Configuration Panel

The screenshot shows the 'Data Configuration' panel of the E(Z)-Kriging software. It features three main visualization areas at the top: a map of sampling points, a semivariogram plot, and a bar chart of Ordinary Point Kriging weights. Below these are three configuration sections: 'data configuration', 'semivariogram definition', and 'Kriging type'. The 'data configuration' section includes a table of data points with a red circle around the 'radius' column.

id	east no.	north no.	radius
0	149	149	37.5
1	124	76	84
2	179	101	5
3	180	156	29
4	112	253	91
5	98	158	36

The values at the sampling locations have to be entered here.

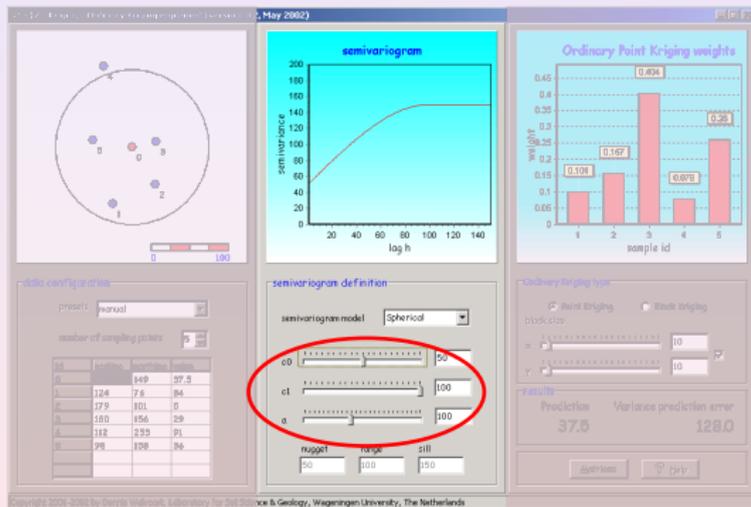
# Semivariogram Panel



The following semivariogram models can be selected:

- Spherical model  
[▶ details](#)
- Exponential model  
[▶ details](#)
- Gaussian model  
[▶ details](#)

# Semivariogram Panel



Use the sliders to set the semivariogram parameters:

- nugget ( $c_0$ )
- partial sill ( $c_1$ )
- range parameter ( $a$ )

# Semivariogram Panel

The screenshot displays the 'Semivariogram Panel' software interface. It is divided into several sections:

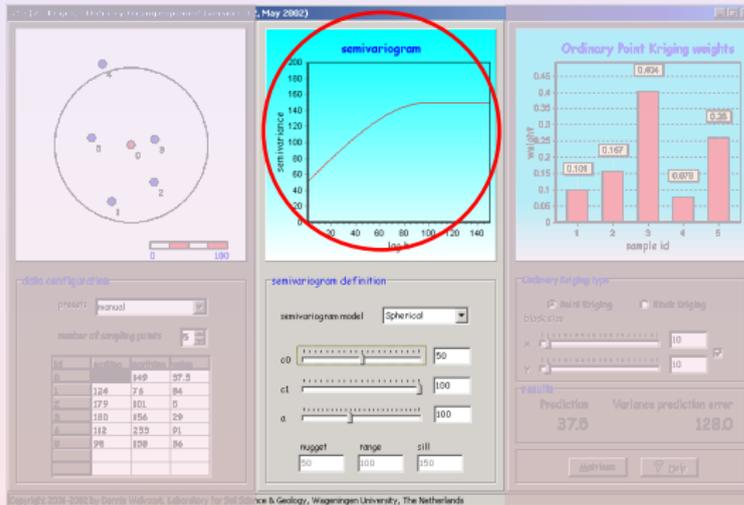
- Top Left:** A map showing the spatial distribution of sampling points (labeled 1-5) within a circular area. A scale bar indicates a distance of 180.
- Top Middle:** A 'semivariogram' plot showing semivariance on the y-axis (0 to 200) versus log h on the x-axis (0 to 140). The curve rises and then levels off, indicating a spherical model.
- Top Right:** A bar chart titled 'Ordinary Point Kriging weights' showing weights for sample IDs 1 through 5. The weights are: 0.109, 0.167, 0.404, 0.075, and 0.243.
- Bottom Left:** A table titled 'data semivariogram' with columns for 'id', 'x', 'y', and 'z'. The data points are:
 

id	x	y	z
1	149	97.8	
2	124	7.6	84
3	179	80.	5
4	180	156	29
5	112	233	91
6	29	108	36
- Bottom Middle:** 'semivariogram definition' section. The model is set to 'Spherical'. Parameters are:
  - c0: 50
  - c1: 100
  - a: 100
  - nugget: 50
  - range: 100
  - sill: 150
 The nugget, range, and sill parameters are circled in red.
- Bottom Right:** 'Ordinary kriging type' section. It shows 'Block size' and 'Number of neighbors' both set to 10. Below, 'Prediction' is 37.5 and 'Variance prediction error' is 128.0.

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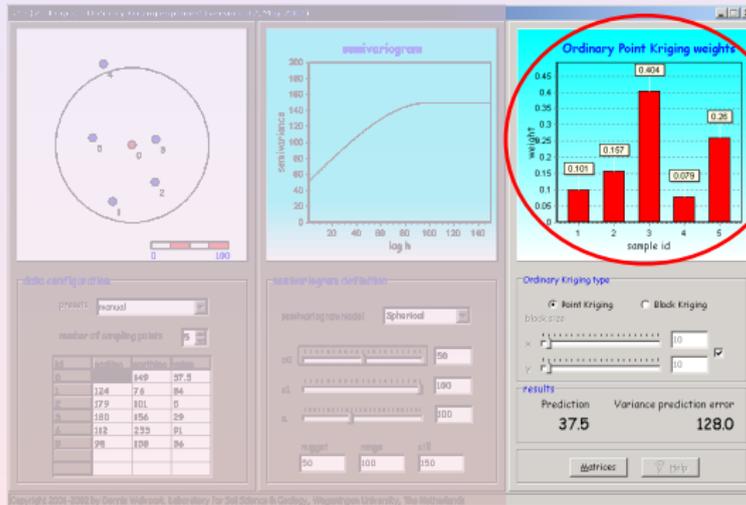
This section gives the nugget variance, the (practical) range and the sill variance ( $c_0 + c_1$ ).

# Semivariogram Panel



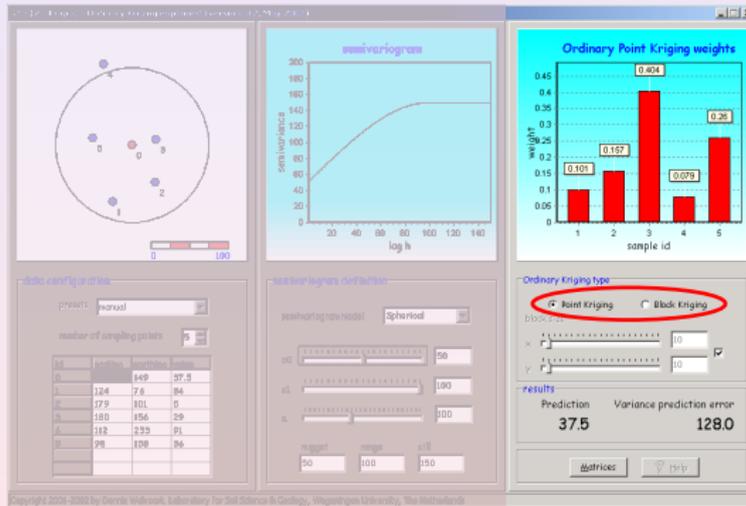
The resulting semivariogram will be shown here.

# Kriging Panel



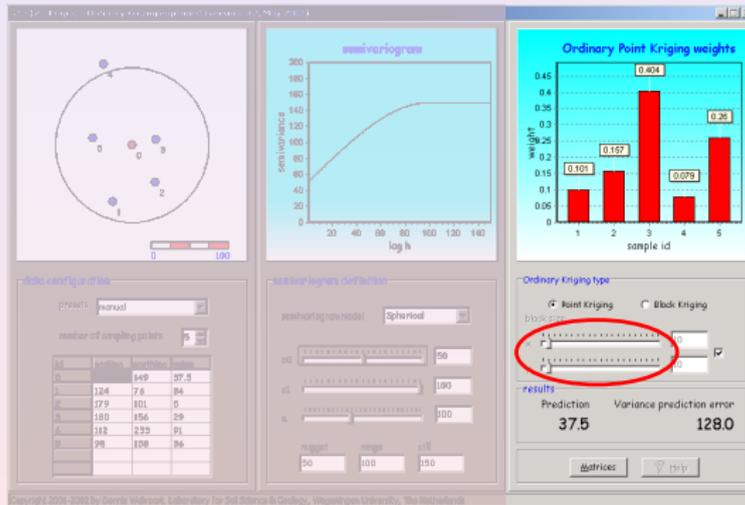
The ordinary kriging weights are shown as bars. Click on the bars for numerical values.

# Kriging Panel



Use the radiobuttons to switch between ordinary point kriging and ordinary block kriging.

# Kriging Panel



The block size can be set by means of the sliders.

# Kriging Panel

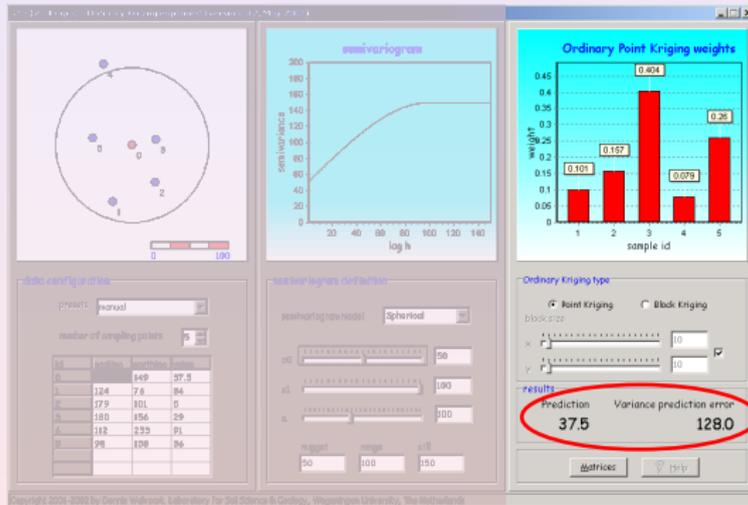
The screenshot displays the Kriging Panel software interface, which is divided into several panels:

- Left Panel:** A map showing the spatial distribution of sampling points (labeled 0, 1, 2, 3, 4) within a circular boundary. A scale bar below indicates a distance of 100 units.
- Semivariogram Panel:** A graph titled "semivariogram" showing the relationship between the logarithm of distance (log h) on the x-axis and semivariance on the y-axis. The curve starts at the origin and levels off, indicating a spherical model.
- Ordinary Point Kriging weights Panel:** A bar chart showing the weights assigned to each of the five sampling points. The weights are: 0.101 (point 1), 0.197 (point 2), 0.404 (point 3), 0.079 (point 4), and 0.26 (point 5).
- Configuration Panels:**
  - Global configuration:** Includes a dropdown menu for "PPSWAT" set to "normal" and a "number of sampling points" set to 5. Below is a table of coordinates for the sampling points.
  - Semivariogram definition:** Includes a dropdown for "semivariogram model" set to "spherical", and input fields for "n0", "n1", "n2", and "n3".
  - Ordinary Kriging type:** Includes radio buttons for "Joint Kriging" and "Block Kriging", and checkboxes for "block size" in the X and Y directions. The "Block Kriging" checkbox is checked, and a red arrow points to it.
  - Results:** Displays "Prediction" as 37.5 and "Variance prediction error" as 128.0.

id	x	y	z
0			149
1	124	7.6	84
2	179	80.	5
3	180	156	29
4	312	233	91
5	398	108	36

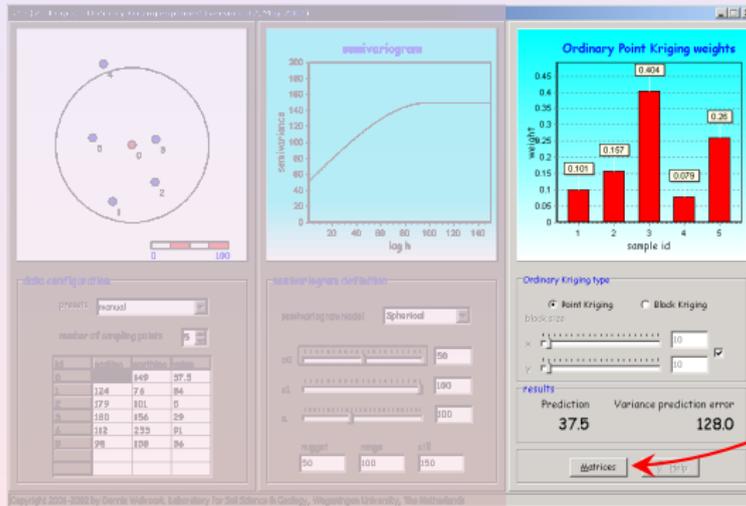
Check this checkbox  
to enforce *square*  
prediction blocks.

# Kriging Panel



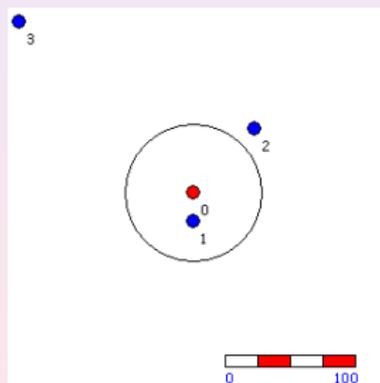
The ordinary kriging prediction and the associated variance of the prediction error are given here.

# Kriging Panel



Press this button to get a glimpse of the underlying maths.

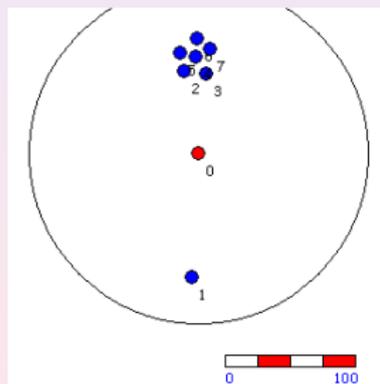
## Interesting features to explore



### Distance effect

Discover that points outside the range affect predictions differently than points within the range (cf. inverse squared distance interpolation).

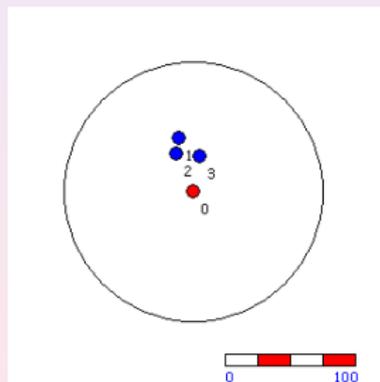
## Interesting features to explore



### Declustering effect

Discover how ordinary kriging reduces the influence of clustered sampling points (cf. inverse squared distance interpolation).

## Interesting features to explore



### Screening effect

Recall that ordinary kriging is a non-convex interpolator, *i.e.*, its predictions can be outside the data range. Explore data configurations and semivariogram settings that enhance this effect.

# Interesting features to explore

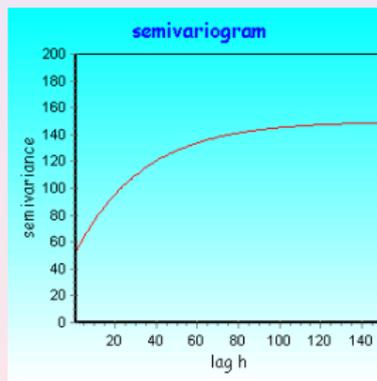
number of sampling points

id	easting	northing	value
0	149	149	24.3
1	133	62	22
2	209	114	88
3	230	208	4
4	158	193	2
5	86	161	14

## Effect of data values

Discover how data values affect the weights, the prediction and the variance of the prediction error.

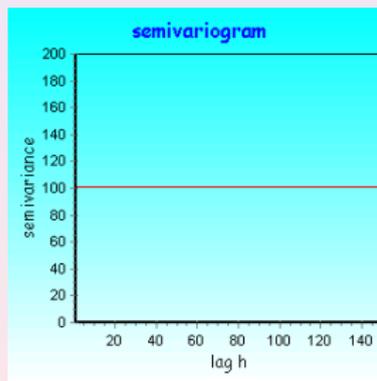
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### Effect of semivariogram model

Study how model shape, nugget, sill and range affect the kriging results.

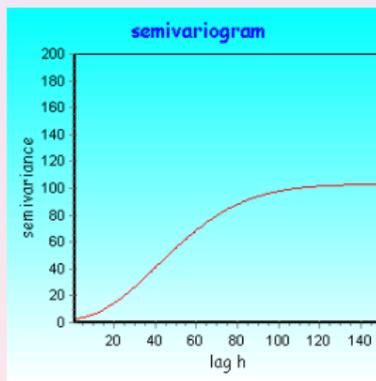
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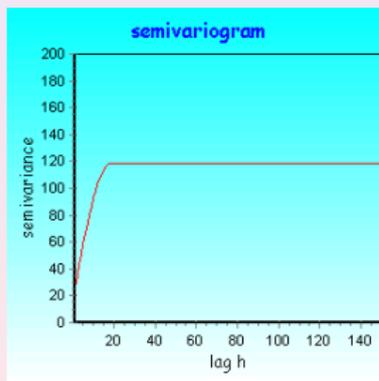
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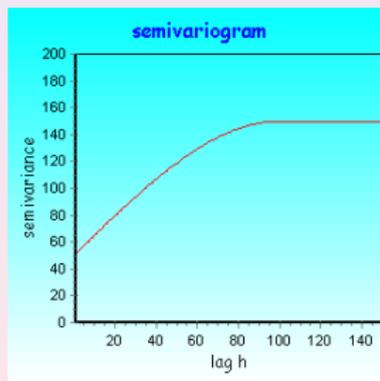
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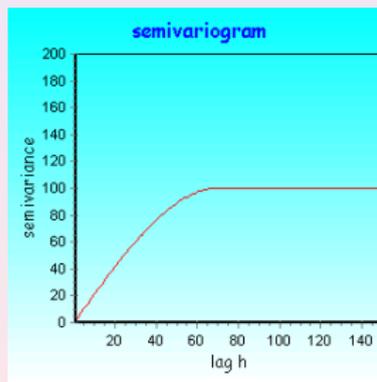
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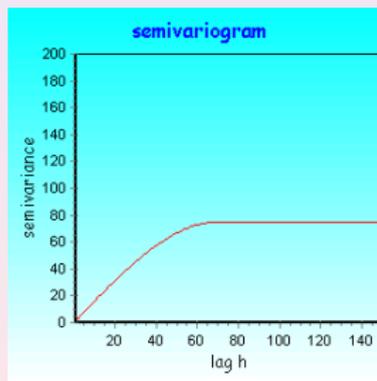
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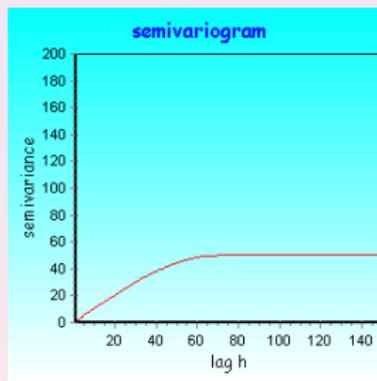
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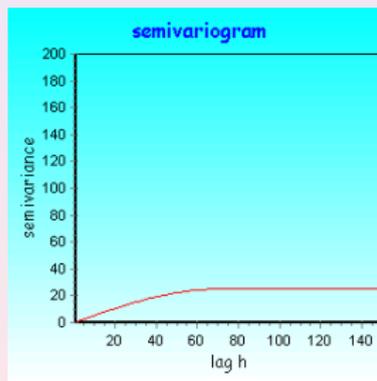
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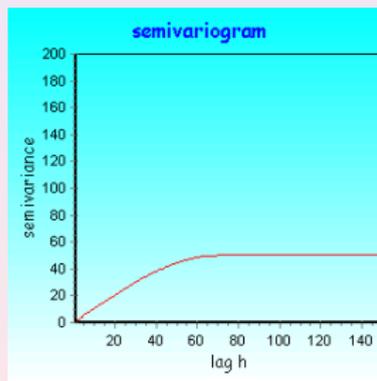
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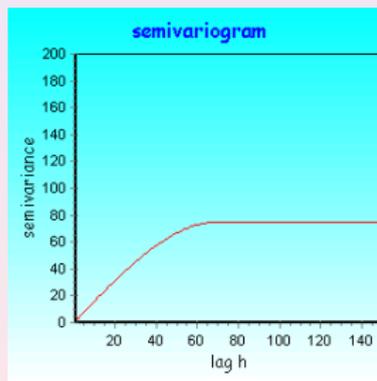
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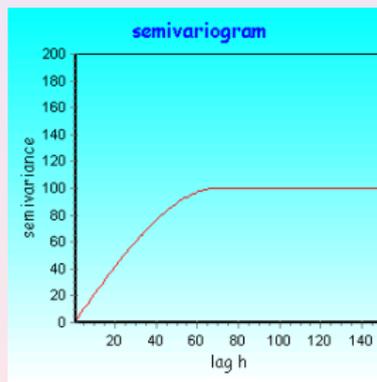
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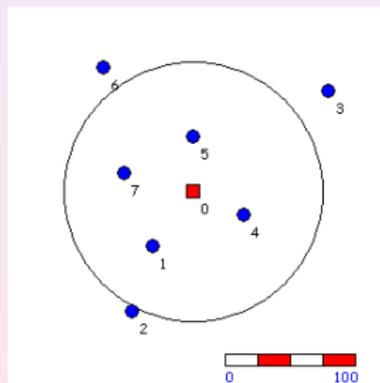
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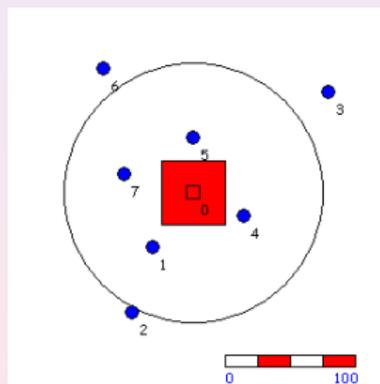
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### Effect of aggregation

Experiment with block size and see how it affects the kriging results.

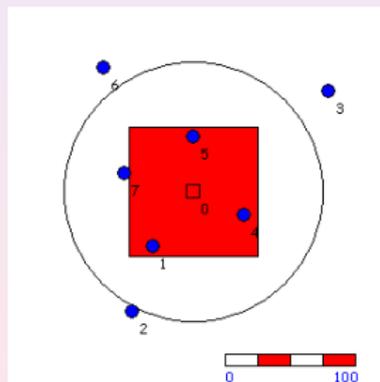
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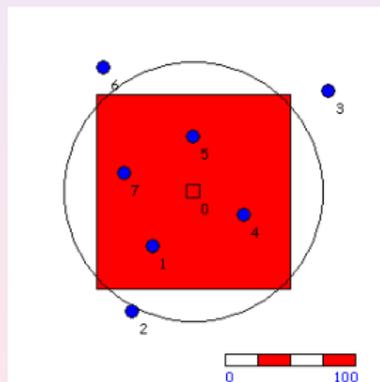
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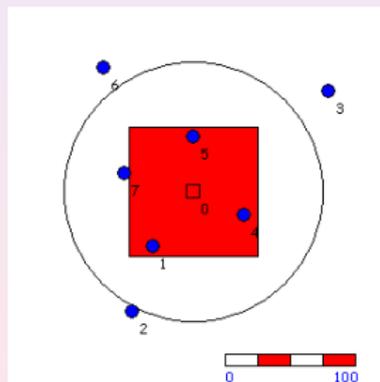
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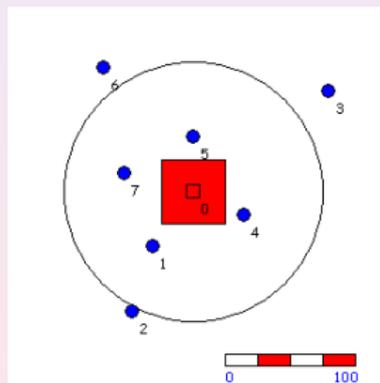
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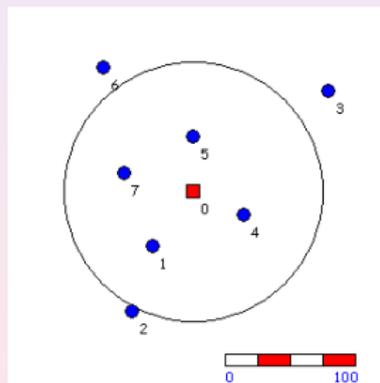
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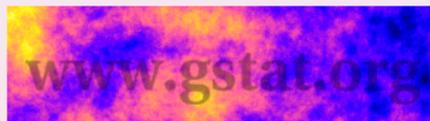
## Suggestions for further reading

-  E.H. Isaaks and R.M. Srivastava.  
*An Introduction to Applied Geostatistics.*  
Oxford University Press, New York, 1989.
-  P. Goovaerts.  
*Geostatistics for Natural Resources Evaluation.*  
Oxford University Press, New York, 1997.

# Geostatistical software

- Vesper ([www.usyd.edu.au/su/agric/acpa/vesper/vesper.html](http://www.usyd.edu.au/su/agric/acpa/vesper/vesper.html))
- GStat ([www.gstat.org](http://www.gstat.org)),  
GStat is also available as R-package ([www.r-project.org](http://www.r-project.org))

Vesper



## License

$E\{Z\}$ -Kriging is freeware and provided *as is* without warranty of any kind, either express or implied.

Enjoy  $E\{Z\}$ -Kriging!

## Spherical model

$$\gamma_s(h) = \begin{cases} \frac{3h}{2a} - \frac{1}{2} \left(\frac{h}{a}\right)^3 & \forall h < a \\ 1 & \forall h \geq a \end{cases}$$

$$\gamma(h) = c_0 + c_1 \gamma_s(h)$$

where:

- $a$  : range parameter
- $c_0$  : nugget variance
- $c_1$  : partial sill variance
- $h$  : lag distance
- $\gamma_s$  : standardised semivariance
- $\gamma$  : semivariance

[Return](#)

## Exponential model

$$\gamma_s(h) = 1 - \exp\left(-\frac{h}{a}\right)$$

$$\gamma(h) = c_0 + c_1\gamma_s(h)$$

where:

$a$  : range parameter

$c_0$  : nugget variance

$c_1$  : partial sill variance

$h$  : lag distance

$\gamma_s$  : standardised semivariance

$\gamma$  : semivariance

◀ Return

## Gaussian model

$$\gamma_s(h) = 1 - \exp\left(-\frac{h^2}{a^2}\right)$$

$$\gamma(h) = c_0 + c_1\gamma_s(h)$$

where:

- $a$  : range parameter
- $c_0$  : nugget variance
- $c_1$  : partial sill variance
- $h$  : lag distance
- $\gamma_s$  : standardised semivariance
- $\gamma$  : semivariance

◀ Return

# Definitions

## Practical range

Lag  $h$  for which  $\gamma(h) = 0.95\gamma(\infty)$ ,  
*i.e.*, that distance at which the semivariance is 95% of the sill.

◀ Return